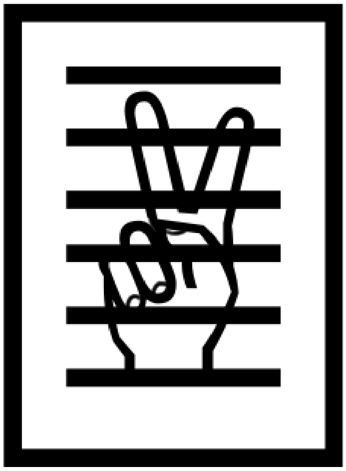
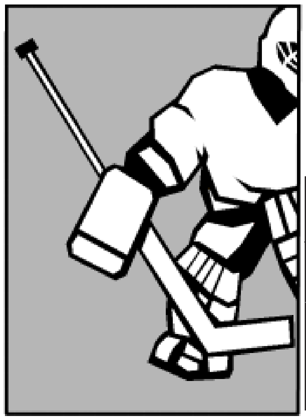
Teaching Problem Solving

Our Biggest Challenge



Teaching Notes

Problem #13



Coach Brice has already chosen six players for his grade four and five inter-school floor hockey team. He has three players who are still trying out for the team, but he is only allowed to choose two more players. Coach Brice looks back at the statistics recorded from previous games and tries to use this data to help him make the wisest choices.

Which two players should Coach Brice choose?

Players	Shots on Goal	Shots Scored
Beth	5	2
Bill	20	5
Brianne	10	3

Floor Hockey Dilemma

Problem #13

The main strategies emphasized in this problem are using kinesthetic visualization and using models or diagrams.

- This problem is appropriate for use with grades four to eight students.
- Begin by asking pairs or groups to discuss which players would prove to be the wisest choices. Allow students some time to reach a collaborative decision. Don't provide feedback or guidance regarding the correct answer. (Students may also wisely point out that ultimately there is no "correct" choice, as any smart coach would always consider a player's demonstrated dependability, leadership skills, ability to play under pressure, and ability to work within a team before he attempted an informed decision.)
- As students mature they tend to employ the following arguments:
    - "Beth would make more goals if she took **10** or **20** shots." (Common denominator)
    - "Bill missed more shots and so did Brianne."
- Many grade-four and five students fall into the seductive trap of selecting Bill and Brianne, as these two players scored more goals. In this case, students have either ignored the ***rate*** of scoring goals or they have perceived the rate as insignificant.
- When developing reasoning skills with students, begin initially by examining Beth's score and introduce the concept of percent. Instruct students to fill in two-fifths of a tenths sheet. (Fifths are created by holding a tenths sheet up to the light and drawing lines on every other visible tenths line. Thus, equal fifths are created.) Students then colour in two of the fifths shown. The sheet is then reversed and students colour in an equivalent area (four-tenths) on the reverse side of their papers. This activity accesses visual memory, which reinforces student learning.
  <u>Note</u>: Eighty-three percent of all learning is acquired visually.
- You may reinforce the concept of equivalent fractions by selecting a group of five students and identifying a common characteristic found within the group: perhaps two of the five are boys. At first two boys and three girls stand in a line at the front of the classroom. Then two more boys and three more girls join the initial group standing at the front.
    - Boy Boy Girl Girl Girl
    - Boy Boy Girl Girl Girl

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- Ask: "What fraction of the students in the front row are boys? (Two fifths.) What fraction in the whole group are boys? (Four tenths.)
- Instruct students to identify the "fraction key" on a calculator. Tell them to search thoroughly. Of course, students are soon delighted to inform you that no such "fraction key" exists. Take this opportunity to convince students that without a handy "fraction key" they must instead redirect their calculators when dealing with fractions. Explain that the negative, or "subtraction" symbol may be employed to execute a division operation in this context. Division is related to repeated subtraction. For example: **5** can be subtracted from **15** three times, as **15** ÷ **5** = **3**. It is essential to represent division equations in their fractional form repeatedly, as it is in precisely this way that most division equations are presented in high school. Ask each student to calculate two-fifths using a calculator. Insist that students employ the "silent mouthing" technique when responding (students mouth the answer but refrain from speaking out loud). Many students will respond with: "Point four." This demonstrates that many students do not fully understand that decimals are fractions. Pronouncing the names of most fractions (fifths, sixths, sevenths, nineteenths, twenty-fourths, twenty-fifths, etc.) elicits a "th" sound. Only the common fraction names such as halves, thirds and quarters do not elicit this sound. Students do not commonly recognize the familiar "th" pattern to fraction names, because most early work with fractions employs only halves, thirds and quarters. The problem examined here presents four-tenths. We initially look at two-fifths rather than four-tenths, as students often are surprised that the **2** and the **5** keys on their calculators yield a puzzling **0.4**. If four-tenths is the first fraction examined, then an erroneous pattern is often perceived: the decimal shown on the calculator is the same as the numerator shown in the fraction. Some students further reinforce this erroneous pattern as follows: when asked to write **2/5** as a decimal, they write **0.25**, clearly indicating that they assume a relationship exists between the numerator, the denominator and the decimal representation that appears on the calculator screen.

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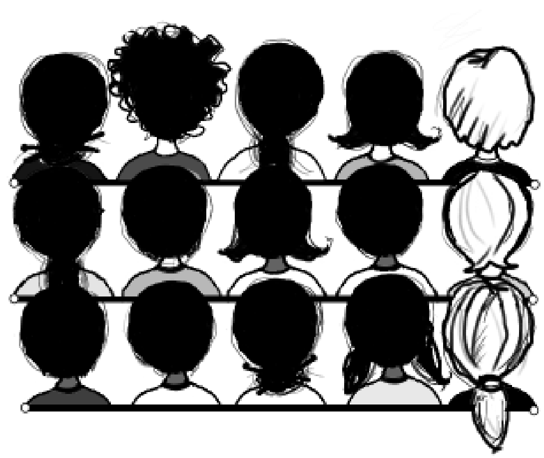
- Establish with students that two-fifths and four-tenths yield the same decimal representation on the calculator. In reality, calculators are designed to present all fractions in tenths wherever possible. This saves energy and time. Calculators are not designed to present users with fifths, sixths, sevenths, elevenths, and twelfths etc.
- However, let us return briefly to our student groups standing at the front of our classroom. Ask two more boys and three more girls to come forward. Students will now be standing in rows as shown:
  - Boy Boy Girl Girl Girl
  - Boy Boy Girl Girl Girl
  - Boy Boy Girl Girl Girl
- Ask: "What fraction of boys are there in the back row? The middle row? The front row? The whole group?" Instruct students to experiment with six-fifteenths on their calculators. Students will be delighted with the surprising **0.4** (pronounced as four-tenths) yielded by their trusty calculators. Students are consistently astonished that the calculator performs so reliably in this way, and they soon develop a sense of equivalent fractions as they persist with the activity. Continue in this way, repeatedly asking two more boys and three more girls to come forward, ultimately yielding (**8/20**, **10/25**, **12/30)**. Consistently reinforce the decimal representation (**0.4)** on the calculator, until eventually you have exhausted your supply of groups of five. A discussion of representative sampling employed during election polling, or sampling pursued by companies eager to establish product preferences, may prove fortuitous at this point.



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- Encouraging students to get out of their seats and to move about creates episodic memory and reinforces that memory kinesthetically. Physical activity engages student attention and provides welcome entertainment and variety. Such activity also involves the whole class in the learning process.
- Parallel the sampling process discussed here by moving to the employment of a hundredths square. Fold the square in half so that the student perceives five rows. Ask each student to colour in two out of five squares in the first row (**2/5**), and then to continue colouring two out of five squares in each subsequent row, thus creating a cumulative effect in which four out of ten squares are coloured as the second row is tackled (**4/10**), and six out of fifteen squares are coloured as the third row is tackled (**6/15**) etc. Relate this activity to the number of successful shots Beth achieved. Establish with students that if Beth successfully makes two out of every five shots, then one may safely assume she would make four out of every ten shots (**4/10**) and six out of every fifteen shots etc. Continue in this way until half the hundredths sheet is shaded (twenty out of fifty squares are shaded). Then instruct students to experiment with **20/50** using their calculators. Students will again be astonished when their calculators yield **0.4**. Consistently reinforce "four-tenths" as a verbal descriptor of **0.4**. Continue with the remaining half of the hundredth sheet. Ask students to unfold their sheets and to note the fractional part that is shaded (**40/100**). Write **100** on the board in the following way: write the **"1"** at a forty-five degree slant and then print two zeros on either side of the slant, thus presenting the "%" symbol. Inform your students that the term "percent" means "hundredths" and that "cent" is the root of words like century, centimetre, centipede, and centurion (all words describing things of **100** parts). Remind students, too, of the one hundred cents in a dollar. Encourage students to view percentage as a fractional representation of **100**, and to view written percentages as another way of writing fractions.

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- Fractions can be written in four ways: common fractions, decimal fractions, percents and ratios. In this lesson all are reinforced. Common fractions are the fractions encountered in everyday life, and decimal fractions are those by which calculators and computers interpret everyday common fractions. Percent relates common fractions to money, and it allows us to compare rates. Ratios are the way in which we talk about sets of items and rates.
- Students often do not understand these relationships because:
  - These concepts are presented as separate units.
  - Students often fail to make the connection between decimals and fractions, as they are rarely encouraged to read decimals as fractions, and instead invariably pronounce **0.4** as "point four".
  - Students are taught rote rules which they cannot later apply in diverse and meaningful contexts.
  - Students are taught decimals within the context of money, and this does not perpetuate a sense of fractions (**$0.27** is rarely viewed as **27/100**).
  - Connections are rarely made between the visual representations of specific fractions and their decimal counterparts.
- Now show Brianne's shots in shaded squares on tenths and hundredths sheets. It will thus become clear to students that Breanne has a **30%** success rate. Ask students to enter the equivalent fractions on their ***Memorable Fractions*** sheets.
- Finally shade Bill's shots on a hundredths sheet. Note that as it is impossible to enter Bill's scores on a tenths sheet, a hundredths sheet must be employed. When students enter Bill's shots they should fold over two columns, or twenty squares, on each hundredths sheet. They continue folding over **20** squares in this way until all twenties have been exhausted. Bill scores at a rate of **25%**. Refrain from relating the **25%** to one-quarter at this point, unless a student does so.
- Encourage students to make the connection between two-fifths and three-fifths. In this example, three out of every five students are girls. Therefore, six out of every ten students are girls, etc. Examine **3/10** and **7/10**: **5/20** and **15/20**. Make connections wherever possible to other events, in order to expand the meaningful context for students in school and community life.

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- "I go to the gym and shoot **15** baskets. I make **6** shots. What percentage did I shoot successfully? What percentage did I miss?" Ask students to calculate these percentages by shading squares on a hundredths sheet, and then to verify their findings on a calculator. Students may find this activity challenging. They must first "block off" rows of **15** squares on a hundredths sheet. When this task has been completed six times, **90** squares will have been accounted for. Students then shade **6** squares out of each group of **15**. The remaining ten squares (beyond the **90**) cannot be employed as fifteenths. However, these ten squares may be regarded as **2** sets of **5** squares. Each set of **5** squares may be viewed as fifteenths, where each individual square is divided into thirds. Students shade **6** thirds of one set of **15** (this actually amounts to **2** squares). They then repeat this process with the remaining **5** squares, thus shading another **2** squares. The total number of squares shaded is ultimately seen as: **6** x **6** + **2** + **2** = **40** squares: or **40%**.
- The Vancouver Canucks win **8** out of **20** games.
  What percentage of games did the Canucks win?
  What percentage did they lose?
- Billy plays Little League baseball. In his last game he went to bat **5** times and made **3** hits.
  What percentage did he hit?
  What percentage did he miss?
- Our school basketball team has won **7** out of **10** games.
  What percentage has the team won?
- I watched the Stanley Cup playoffs last night. In the game Edmonton shot **20** times and made **5** goals, Calgary shot **24** times and made **6** goals. Calgary won.
  Which team had the best shooting percentage?
- Jeremy got **15** out of **20** on his spelling test.
  What percentage did he spell correctly?
  What percentage did he spell incorrectly?
- Students enter all fractions generated on a ***Memorable Fraction*** sheet and place the sheet at the back of their exercise books. Make an effort to focus upon a fraction/decimal/percent problem at least once a week with students. Note that you have an opportune moment to do this whenever a class test is scored out of a possible **5**, **10**, **20**, **25**, **50** or **100** marks. Students are now able to calculate their own percentages when their assignments have been graded.

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- This lesson may well cover two or three periods in a grade-four class, but the concepts could be adequately covered in only one period with grade-eight students.
- It is essential that you revisit the **"Memorable Fractions"** sheet every week. Begin examining related fractions and percentages at the outset of the school year. Students will then develop a strong sense of decimal fractions, which can be integrated into addition, subtraction, multiplication and division activities in class.
- Eventually the connection should be made to repeating and non-repeating fractions and their decimal equivalents. Ultimately, students should be able to predict before entering a fraction on their calculator whether or not that fraction will yield a repeating decimal fraction.